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# COMPRESSIVE ONLINE ROBUST PRINCIPAL COMPONENT ANALYSIS WITH MULTIPLE PRIOR INFORMATION

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## 1. Motivation

- Applications: Computer vision, web data analysis, anomaly detection, and data visualization, etc.
- Robust Principal Component Analysis (RPCA): Batch-based, decomposes all data samples (matrix  $M$ ) into low-rank ( $L$ ) and sparse ( $S$ ), e.g., all frames in a video, high computational and memory requirements

$$\min_{L,S} \|L\|_* + \lambda \|S\|_1 \text{ subject to } M = L + S$$

### Challenges

- Online method processing a sequence of signals per time instance from a small set of measurements  $y_t = \Phi(x_t + v_t)$

$$M_t = L_t + S_t \text{ into } S_t = [x_1 \ x_1 \dots x_t] \text{ and } L_t = [v_1 \ v_2 \dots v_t]$$

### ➤ Minimization at time instance $t$

$$\min_{x_t, v_t} \|[L_{t-1} \ v_t]\|_* + \lambda \|x_t\|_1 \text{ subject to } y_t = \Phi(x_t + v_t)$$

where  $L_{t-1} = [v_1 \ v_2 \dots v_{t-1}]$ ,  $S_{t-1} = [x_1 \ x_1 \dots x_{t-1}]$ ,  $\Phi$  are given

## 2. Compressive Online RPCA (CORPCA) With Multiple Prior Information

### Problem formulation

- Incorporating multiple prior information: at time instance  $t$  we observe  $y_t = \Phi(x_t + v_t)$  with  $y_t \in \mathbb{R}^m$  given priors  $Z_{t-1}$  and  $B_{t-1}$  from  $\{\hat{x}_1, \dots, \hat{x}_{t-1}\}$  and  $\{\hat{v}_1, \dots, \hat{v}_{t-1}\}$

- Solving the  $n$ - $\ell_1$  minimization problem

$$\min_{x_t, v_t} \left\{ H(x_t, v_t | y_t, Z_{t-1}, B_{t-1}) = \frac{1}{2} \|\Phi(x_t + v_t) - y_t\|_2^2 + \lambda \mu \sum_{j=0}^J \beta_j \|\mathbf{W}_j(x_t - z_j)\|_1 + \mu \|[B_{t-1} \ v_t]\|_* \right\},$$

where  $\lambda > 0$  and  $\beta_j > 0$  are weights across the side information signals, and  $\mathbf{W}_j$  is a diagonal matrix with weights for each element in the side information signal  $z_j$ ; namely,  $\mathbf{W}_j = \text{diag}(w_{j1}, w_{j2}, \dots, w_{jn})$  with  $w_{ji} > 0$  being the weight for the  $i$ -th element in the  $z_j$  vector.

### The CORPCA algorithm

- Solving  $n$ - $\ell_1$  minimization via the soft thresholding operator and the single value thresholding operator, at iteration  $k + 1$

$$\begin{aligned} v_t^{(k+1)} &= \arg \min_{v_t} \left\{ \mu h(v_t) + \left\| v_t - \left( v_t^{(k)} - \frac{1}{2} \nabla_{v_t} f(v_t^{(k)}, x_t^{(k)}) \right) \right\|_2^2 \right\} \\ x_t^{(k+1)} &= \arg \min_{x_t} \left\{ \mu g(x_t) + \left\| x_t - \left( x_t^{(k)} - \frac{1}{2} \nabla_{x_t} f(v_t^{(k)}, x_t^{(k)}) \right) \right\|_2^2 \right\} \end{aligned}$$

$$\text{where } f(v_t, x_t) = (1/2) \|\Phi(x_t + v_t) - y_t\|_2^2$$

$$g(x_t) = \lambda \sum_{j=0}^J \beta_j \|\mathbf{W}_j(x_t - z_j)\|_1, \text{ and } h(v_t) = \|[B_{t-1} \ v_t]\|_*$$

- Updating weights  $\beta_j$  and  $\mathbf{W}_j$

- After solving for time instance  $t$ : Prior updates

$$Z_t := \{z_j = x_{t-J+j}\}_{j=1}^J$$

$$B_t = U_t(:, 1:d) \Gamma_{\frac{\mu_k}{2} g_1}(\Sigma_t)(1:d, 1:d) V_t(:, 1:d)^T$$

Algorithm 1: The proposed CORPCA algorithm.

```

Input:  $y_t, \Phi, Z_{t-1}, B_{t-1}$ ;
Output:  $\hat{x}_t, \hat{v}_t, Z_t, B_t$ ;
// Initialize variables and parameters.
1  $x_t^{(-1)} = x_t^{(0)} = 0; v_t^{(-1)} = v_t^{(0)} = 0; \xi_{-1} = \xi_0 = 1; \mu_0 = 0;$ 
 $\bar{\mu} > 0; \lambda > 0; 0 < \epsilon < 1; k = 0; g_1(\cdot) = \|\cdot\|_1;$ 
2 while not converged do
    // Solve Problem (4).
3  $\tilde{v}_t^{(k)} = v_t^{(k)} + \frac{\xi_{k-1}-1}{\xi_k} (v_t^{(k)} - v_t^{(k-1)});$ 
4  $\tilde{x}_t^{(k)} = x_t^{(k)} + \frac{\xi_{k-1}-1}{\xi_k} (x_t^{(k)} - x_t^{(k-1)});$ 
5  $\nabla_{v_t} f(\tilde{v}_t^{(k)}, \tilde{x}_t^{(k)}) = \nabla_{v_t} f(v_t^{(k)}, x_t^{(k)}) =$ 
 $\Phi^T (\Phi(\tilde{v}_t^{(k)} + \tilde{x}_t^{(k)}) - y_t);$ 
6  $(U_t, \Sigma_t, V_t) =$ 
 $\text{incSVD}([B_{t-1} \ (\tilde{v}_t^{(k)} - \frac{1}{2} \nabla_{v_t} f(\tilde{v}_t^{(k)}, \tilde{x}_t^{(k)}))]);$ 
7  $\Theta_t = U_t \Gamma_{\frac{\mu_k}{2} g_1}(\Sigma_t) V_t^T;$ 
8  $v_t^{(k+1)} = \Theta_t(:, \text{end});$ 
9  $x_t^{(k+1)} = \Gamma_{\frac{\mu_k}{2} g}(\tilde{x}_t^{(k)} - \frac{1}{2} \nabla_{x_t} f(\tilde{v}_t^{(k)}, \tilde{x}_t^{(k)}));$  where
 $\Gamma_{\frac{\mu_k}{2} g}(\cdot)$  is given as in RAMSIA [18];
// Compute the updated weights [18].
10  $w_{ji} = \frac{n(|x_{ji}^{(k+1)} - z_{ji}| + \epsilon)^{-1}}{\sum_{i=1}^n (|x_{ti}^{(k+1)} - z_{ji}| + \epsilon)^{-1}}$ 
11  $\beta_j = \frac{\left( \|\mathbf{W}_j(x_t^{(k+1)} - z_j)\|_1 + \epsilon \right)^{-1}}{\sum_{i=0}^J \left( \|\mathbf{W}_i(x_t^{(k+1)} - z_i)\|_1 + \epsilon \right)^{-1}};$ 
12  $\xi_{k+1} = (1 + \sqrt{1 + 4\xi_k^2})/2; \mu_{k+1} = \max(\epsilon \mu_k, \bar{\mu});$ 
 $k = k + 1;$ 
14 end
// Update prior information.
15  $Z_t := \{z_j = x_{t-J+j}^{(k+1)}\}_{j=1}^J;$ 
16  $B_t = U_t(:, 1:d) \Gamma_{\frac{\mu_{k+1}}{2} g_1}(\Sigma_t)(1:d, 1:d) V_t(:, 1:d)^T;$ 
17 return  $\hat{x}_t = x_t^{(k+1)}, \hat{v}_t = v_t^{(k+1)}, Z_t, B_t;$ 

```

## 3. Experimental Results

### Synthetic data

- Generating low-rank components:  $n = 500$ ,  $d = 100$  (training),  $q = 100$  (testing),  $r = 5$  (rank)  
 $L = UV^T$ , where  $U \in \mathbb{R}^{n \times r}$  and  $V \in \mathbb{R}^{(d+q) \times r}$  yields  $L = [v_1 \dots v_{d+q}]$
- Generating sparse components with  $\|x_t\|_0 = s_0$  and  $\|x_t - x_{t-1}\|_0 = s_0/2$  obtaining  
 $S = [x_1 \dots x_{d+q}]$
- Testing on  $M = [x_{d+1} + v_{d+1} \dots x_{d+q} + v_{d+q}]$
- Measuring probabilities of successful decomposition,  $\text{Pr}(\text{success})$ , success if  $\|\hat{x}_t - x_t\|_2 / \|x_t\|_2 \leq 10^{-2}$

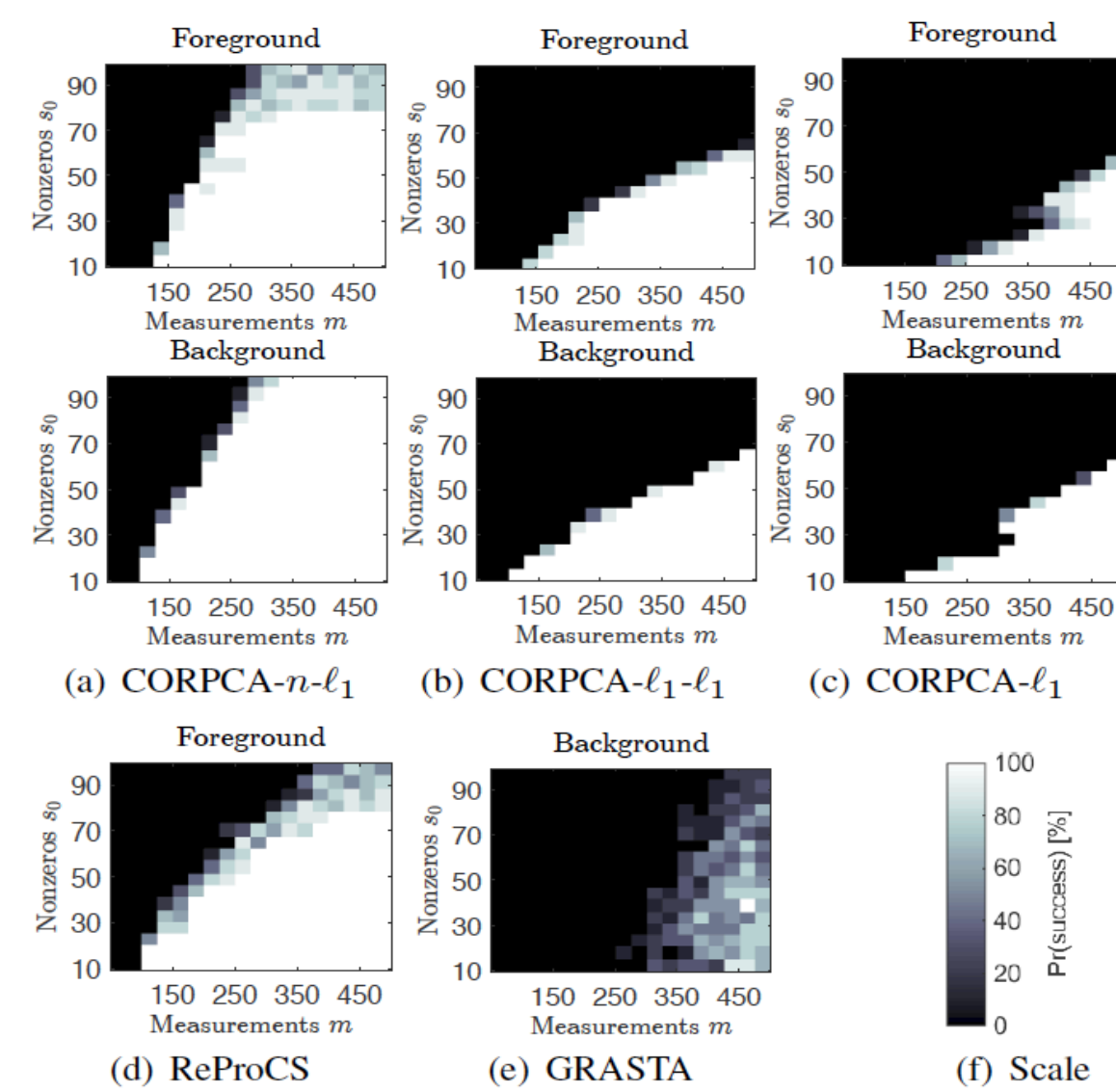


Fig. 1. Average success probabilities for CORPCA (for  $x_t, v_t$ ), ReProCS (for  $x_t$ ), and GRASTA (for  $v_t$ ). The scale (f) is proportional to  $\text{Pr}(\text{success})$  [%] from black to white.

### Compressive video foreground-background separation

- Considering two videos, Bootstrap (60x80 pixels) and Curtain (64x80 pixels) having a static and a dynamic background, respectively
- Background-foreground video separation with full access to the video data
- Compressive separating by varying rates on the number of measurements  $m$  over the dimension of the data  $n$

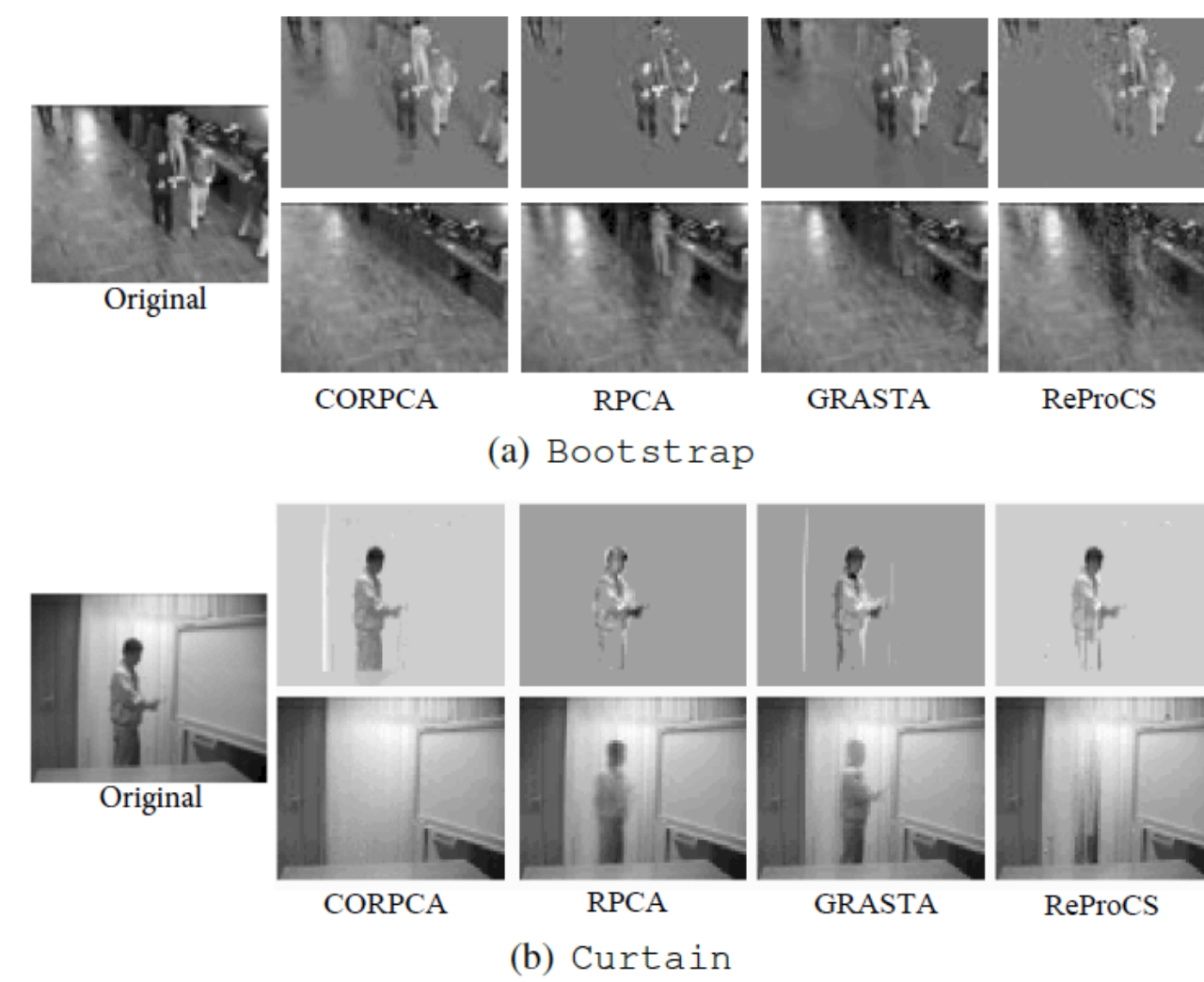


Fig. 2. Background and foreground separation for the different separation methods with full data access Bootstrap#2213 and Curtain#2866.

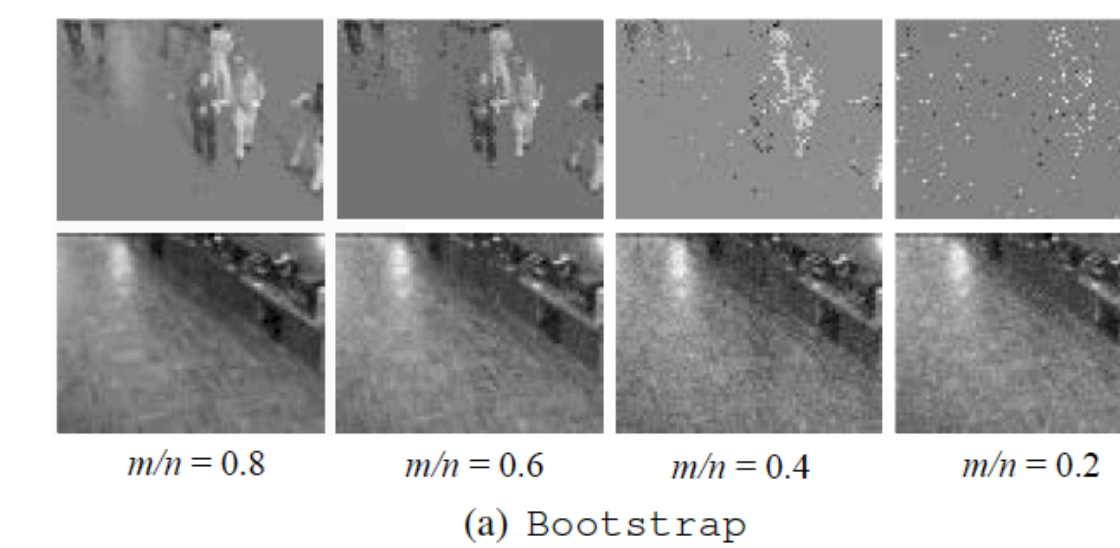


Fig. 3. Compressive background and foreground separation of CORPCA with different measurement rates  $m/n$ .

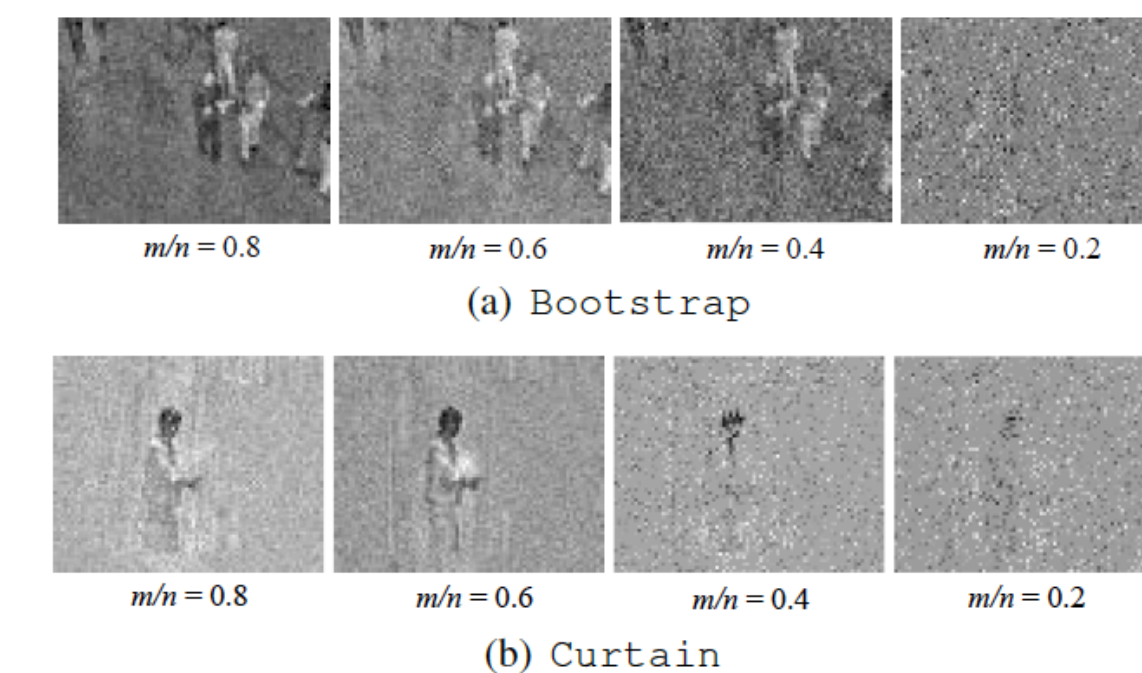


Fig. 4. Compressive foreground separation of ReProCS with different measurement rates  $m/n$ .

## 4. Summary

### Solution for an $n$ - $\ell_1$ minimization

- Incorporating efficiently multiple prior information
- Updating iteratively weights

### The proposed COPRCA algorithm

- Processing a data vector per time instance using compressive measurements
- Solving the  $n$ - $\ell_1$  minimization and updating priors for the next instance

### Evaluation of COPRCA on synthetic data and actual video data

- Outperforming classical compressive sensing (CS) ( $\ell_1$  minimization) and CS with single prior information ( $\ell_1$ - $\ell_1$  minimization)
- The superior performance improvement compared to the existing methods

CORPCA source code, test sequences, and the corresponding outcomes. [Online]. Available: <https://github.com/huynhvlvd/corpcap>